1c) The procedure of Erbelin et al approach to obtain the pdf of log returns under the Variance Gamma process. Uses the characteristic function of the variance Gamma process which we have coded in the function titled vgchar. In order to obtain the density function using the quadrature (integral) implementation of Eberlin we coded the function fourier\_vg\_char. This recovered the risk neutral drift mu using the log of the characteristic function.

We then simulated the integrand of the pdf taking the characteristic function and multiplying it by the fourier transform of the payoff of the option given by the formula . We then computed the integral of the real part of this formula and multiplied it by the remaining fourier transform of the strike price and the dampening factor required which is needed to ensure the function is Lebasque integrable  (*The Lebesgue integral(no date.)* We then created 3-d interactive plots in MATLAB using a modified function taken from (Kienitz Wetterau FinModelling (2023) )matlab code.

We see an obvious pattern with the prices of call options increasing quickly as the strike price decreases and prices of options doing the opposite intersection at a strike price of approximately 105. As the time to maturity increases there is a greater time volatility component however the effect is much weaker than the strike price variation. The price in relation to the strike price is asymmetric for this data range with the price of the call rising faster than the price of the put decays suggesting that the volatility of the call option is greater more upside potential and therefore a higher price is required to maintain no arbitrage in the market. Run the code in the first MATLAB section to see full data set. When we extend the ranges of the strike price we get a classic volatility smile where implied volatility is plotted against the strike price Mitchell, C. (2022.)

Chart

Description automatically generatedChart

Description automatically generated

1d) We used the Black Scholes implied volatility function ‘blsimpv’ in matlab on our options pricing obtained from the Variance Gamma process to extract the implied volatility. This is a numerical method of fitting the observed price to the implied volatility of the BS formula. This volatility is what the market prices in for the option, the expectation of volatility risk over the lifetime of the option. For each different parameter estimate required we created a new matrix of prices where the rows represented the time to maturity and the columns the strike prices. It is easy to modify this to obtain any desired range. We performed this on both calls and puts to create a matrix of data for the varying parameter estimates over varying strike prices and maturities.

1e) ITM: In the money, ATM: At the money, IV=Implied volatility.

These plot are interactive in MATLAB. Having the data in matrices we plotted IV on the z axis against time to maturity and strike price ranges with the different values of parameters set against the base parameter to examine how to the IV changed with respect to the factors involved.

Under the Black-Scholes model we expect implied volatility to be the same across calls and puts due to extracting the volatility under the assumption of put-call parity. For the same strike price and maturity the IV for puts and calls must be the same under the Black-Scholes model (True Tamplin, B.S. (2023.)) As can be seen from below where the graphs overlap completely. This also acts as a baseline.

Chart, surface chart

Description automatically generated

Mu:

Chart, surface chart

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Mu functions as location parameter and we can see that it is roughly symmetric in our sensitivity analysis as seen in the left most graph is symmetric along the x axis with the negative simply inverting the curve and the size determining how far it is from low volatility as high mu implies being further from the strike price. As mu -1 and 1 are further apart on the number line they intersect less as the higher the mu parameter the further it is from the central strike price and being more ITM creates more implied volatility in the option price.

Theta:

Chart

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Theta is also symmetrical around the x axis with the negative and positive values perfectly overlapping. It has a huge effect on volatility but is almost invariant under time to maturity and strike price implying it is simply a scaling parameter. This can be seen in the characteristic function we integrate over as it has a linear dynamic. If it has an influence on the skew (week 5 tutorial) then this means that extra weight is given to one side ITM scaling the IV by shifting the weight of the pdf of the Variance Gamma function.

Sigma:

Chart, surface chart

Description automatically generatedChart, surface chart

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Sigma creates an extremely steep volatility smile reflecting on the variance of the distribution which controls how likely the distribution is the to head into ITM area’s however, as sigma increases the volatility increases rapidly as investors go long volatility in options under the BS model *8.4 the black-Scholes model* and higher volatile will exponentially draw investor interest(no date.)

Kappa:

Chart

Description automatically generatedChart, surface chart

Description automatically generatedChart, surface chart

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Kappa has a strong left side bias reflecting its right limiting distributional properties of the pdf generated in 1b. This could imply a kurtosis effect with kappa creating a heavier left tail of the distribution creating higher volatility at lower strike prices as investors long volatility.

All models can be seen to have their implied volatility spike as time to maturity approached zero as the approaching expiration date means the payoff is closer investors will seek to balance their share of the underlying asset and the option in their portfolio to delta-gamma hedge their risk and cover themselves leading to fluctuations in the price and therefore more volatility implied in the option (Gort, C. and Burgener, E. (March 2014)). Additionally, for all parameter sets the further you get from the strike price in either direction will lead to high implied volatility, previously referred to as a volatility smile as the more ITM and OTM an option is the more its price is likely to fluctuate. Sigma seems to have the greatest impact on strike price.

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